**The Time Value of Money**

The time value of money - The most basic concept in Finance.

**Definitions:**

**Interest** - Money paid for the use of your money. Expressed as a % or a decimal.

**Future Value** - Amount to which an investment will grow after earning interest.

**Present Value** – Amount of money you start with – the initial investment.

**Simple Interest** - Interest earned on the original investment

**Compound Interest** - Interest earned on interest

**Example**: Interest = Interest Rate x Present Value (your initial investment)

 $6 = .06 · $100

Value after one year = $100 + $6 = $106

Let r = interest rate; PV = Present Value and FV = Future Value

Value after one year = PV (1+r)

 $100 (1 + .06) = $106

Second Year - Start with $106

 Interest = $106 ·.06 = $6.36

Value at end of year = $106 + $6.36 = $112.36

 = $106 (1 + r)

 = $106 (1 + .06) = $112.36

Start at beginning to year one and go to the value at the end of year 2.

100\_\_\_\_\_\_\_\_\_\_\_106\_\_\_\_\_\_\_\_\_\_\_\_\_112.36

 0 1 2

Value after 2 years = $100 (1.06) (1.06) = $112.36

 = $100 (1.06)2 = $112.36

 = PV (1+r) (1+r)

 = PV (1+r)2

FV = PV (1+r)t

where t = number of compounding periods

and r = interest rate per compounding period

Taking it out to 5 years:

100\_\_\_\_\_\_106\_\_\_\_\_\_\_112.36\_\_\_\_\_\_\_119.10\_\_\_\_\_\_\_\_126.25\_\_\_\_\_\_\_133.82

 0 1 2 3 4 5

FV = PV (1 + r)t Here r = .06 = 6%

for t = 3 FV = 100 (1.06)3 = $119.10

for t = 4 FV = 100 (1.06)4 = $126.25

for t = 5 FV = 100 (1.06)5 = $133.82

On **Excel**:

Locate Future Value with the Function Wizard

1. Enter .06 or 6% as Rate
2. Enter 5 as Nper
3. Leave Pmt blank
4. Enter 100 as PV
5. Leave Type blank
6. Note that if you enter PV as a positive value, FV will be negative

**Another example**: Invest $25 for 2 years at 9%. What will it grow to?

FV = PV (1+r)t

 = $25 (1.09)2

 = $25 (1.09) (1.09) = $29.70

Excel: FV function

Enter .09 or 9% as Rate

Enter 2 as Nper

Leave Pmt blank

Enter 25 as PV

Leave Type blank

If you want to see FV as a positive value, either enter -25 for PV or solve for -FV

 25 (1.09) 25 (1.09)2

25 27.25 29.70

 0 1 2

Try $10 at 5% for 30 years

FV = PV (1+r)t

 = 10 (1.05)30

 = 10 (4.3219) = $43.22

***Problem***: If you invest $1,000 for 35 years at 6% interest, what will it grow to?

Note that t can be any time period (month, week, year, quarter, etc.) – it is the number of compounding periods and r is the interest rate **per compounding period**.

**Compounding Period**: How often interest is posted. Immediately after it is posted, you start earning (or paying) interest on the interest.

**Example**: Credit cards: Interest accrues monthly

 Monthly rate = 1.5%

 You charge $100

 You wait 2 years to pay it off - what do you owe?

FV = PV (1+r)t where r = 1.5% (interest rate per month) and t = 24 months

 = 100 (1.015)24

 = $142.95 = $100 principal plus $42.95 interest

Question: If 1.5% = monthly rate, what is the annual rate?

**APR =** **Annual Percentage Rate:** the most commonly used way to express interest rates.

APR = r · m where r = int. rate per compounding period and m = number of compounding periods in a year.

1.5% / month = 18% APR because (1.5) (12) = 18.

APR is the most commonly used interest rate. Whenever you see an interest rate, you should assume it is the APR unless it is specified otherwise.

APR/m (where “m” is the number of compounding periods in a year) gives you the interest rate per compounding period because APR is found by multiplying the interst rate for the compounding period by m.

**Example**: If the interest rate on your car loan is quoted to you as 7.5%, they are quoting you the APR. If you make monthly payments, the monthly interest rate is not 7.5%, it is 7.5%/12 = 0.625% per month.

***Problem***: You go to a bank that advertises an interest rate (APR) of 4% on your deposits with daily compounding. If you deposit $500 into the bank, how much will you have in your account at the end of 200 days?

We can take the equation FV = PV (1+r)t and rearrange it to get

PV = FV\_\_\_

 (1+r)t

PV = value today of a future cash flow

r = discount rate (interest rate)

To calculate PV we discount FV at interest rate r over t periods

**Example**: How much to invest today for it to grow to $500 in 2 years if interest rate is 7%?

PV = \_FV\_\_

 (1+r)t

 = 500 .

 (1.07)2

 = 436.72

Invest $436.72 today at 7% to get $500 in 2 years.

436.72\_\_\_\_\_\_\_\_\_467.29\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_500.00\_\_

 0 1 2

**Another Example**:

What is the value today of $50,000 to be received in 10 years if interest rate is 9% (APR) and we have monthly compounding?

Excel: Find PV under function wizard

Rate = .09/12

Nper = 10 x 12

Pmt = blank

Fv = 50,000

Type = blank

PV = \_FV\_

 (1+r)t

 = 50,000

 

 = 20,396.87

***Problem***: If you need $30,000 in 5 years, how much must you invest today, if the rate of return on your investment is 6% (APR) with monthly compounding?

So far, we have 2 basic equations:

1. FV = PV (1+r)t

2. PV = \_\_FV\_\_

 (1+r)t

Suppose we want to find r (the interest rate per compounding period) or t (the number of compounding periods)?

In Excel, r = RATE and t = Nper

Suppose someone tells you if you give him $100 today, he’ll give you $120 in 3 years. Is this a good deal or a bad deal? First we must determine what interest rate we are getting.

But note: To solve this, we must make an assumption about how frequently our money is compounded. Let’s assume annual compounding.

PV = $100 FV = $120 t = 3 years (annual compounding)

100 120

 0 1 2 3

Using Excel: Find Rate under the function wizard

Nper = 3

Pmt = blank

Pv = -100

Fv = 120

Type = blank

Rate = 6.27% Now that I know this, I can more easily evaluate whether this is a good investment or not.

***Problem***: If you borrow $1,000 today and must pay back $1,100 in one month, what is the interest rate you are paying – expressed as an APR?

**Solving for t:**

**Example**:

How long will it take a $1,000 investment to grow to $100,000 if it earns 14% per year (annual compounding)?

 1,000 100,000

 0 ?

Excel: Find Nper on function wizard

Rate = .14

Pmt = blank

Pv = -1000

Fv = 100000

Type = blank

Nper = 35.146

Note that if interest is posted at the end of the year, you won’t have it until the end of year 36!

**Summing the Present Values**

You are the agent for a professional athlete. Two contract alternatives are presented to you. Which is better?

These are the cash flows your client is guaranteed to receive:

Contract A: $1 million per year for five years with each payment coming at the end of the year.

Contract B: $3.1 million at the end of the first year and $400,000 at the end of each of the following four years.

Under contract A, your athlete will receive a total of $5 million

Under contract B, your athlete will receive a total of $4.7 million

Unfortunately, you can’t add money received in different time periods - even if they are paired up in the same years like this.

You must get all the money valued as of one time period. We’ll use the present.

If r = 10%

1. $1 mill. = $909,091 B) $3.1 mill = $2,818,182

 (1.1)1 (1.1)1

 $1 mill = $826,446 $400,000 = $330,579

 (1.1)2 (1.1)2

 $1 mill = $751,315 $400,000 = $300,526

 (1.1)3 (1.1)3

 $1 mill = $683,013 $400,000 = $273,205

 (1.1)4 (1.1)4

 $1 mill = $620,921 $400,000 = $248,369

 (1.1)5 (1.1)5 .

 $3,790,786 $3,970,860

So if r = 10%, PVB > PVA

**Excel**: Find NPV on function wizard

Rate = .10

Values must be entered as: 1,000,000, 1,000,000, 1,000,000, 1,000,000, 1,000,000

If r = 2%

A) $1 mill. = $980,392 B) $3.1 mill = $3,039,216

 (1.02)1 (1.02)1

 $1 mill = $961,169 $400,000 = $384,468

 (1.02)2 (1.02)2

 $1 mill = $942,322 $400,000 = $376,929

 (1.02)3 (1.02)3

 $1 mill = $923,845 $400,000 = $369,538

 (1.02)4 (1.02)4

 $1 mill = $905,731 $400,000 = $362,292

 (1.02)5 (1.02)5 .

 $4,713,460 $4,532,443

So if r = 2% then PVA > PVB

Notice that the higher the interest rate, the more important it is to get money early on.

The main point is that if you wish to compare money received in different time periods, you must first convert all CF to PV.

You can always add PVs but not money in different time periods.

***Problem***: If interest rates are 5%, which is the best contract?

Interest Rate is 5%. What is the total PV of: $1,000 at end of year 1,2,3,4 &5?

PV = $1,000 + $1,000 + $1,000 + $1,000 + $1,000

 1.05 1.052 1.053 1.054 1.055

 = 952.38 + 907.03 + 863.84 + 822.70 + 783.53

 = $4,329.48

Note that we can now introduce the Pmt function in excel

Excel: Find PV on function wizard

Rate = .05

Nper = 5

Pmt = 1,000

Fv = blank

Type = blank

**Here’s another way to look at it:**

$4,329.48 will provide $1,000/year for 5 years if the interest rate is 5%.

Year Start Interest Earned Total Payout Remainder

1 $4,329.48 $216.47 $4545.95 $1000 $3545.95

2 $3,545.95 177.30 3723.25 1000 2723.25

3 $2,723.25 136.16 2859.41 1000 1859.41

4 $1,859.41 92.97 1952.38 1000 952.38

5 $ 952.38 47.62 1000.00 1000 0

So, at 5%, how much do we need to provide $1,000/year **forever**?

If it’s to go forever, we can’t use up any principal, so, we must produce $1,000 in interest each year.

# At 5%, how much principal is needed to produce $1,000 in interest?

Interest = Interest Rate x Principal

Principal = \_\_\_Interest\_\_\_ = $1,000 = $20,000

 Interest Rate .05

So $20,000 is the principal needed to produce $1000 interest at 5%

$20,000 · 5% = $1,000 - Pay the $1,000 and start over each year.

**Perpetuity** - A stream of level cash flows that never ends.

PV of Perpetuity = Cash Flow or, PVP = C

 Int. Rate r

**Annuity** - Stream of level cash payments that ends. It’s not an annuity if the payments are not all equal.

PVA =  The expression in brackets is called the annuity factor.

From our previous example:PVA = 

 = $4,329.48 which is the same result we got previously

Example: State lottery: How much is $50,000/year for 20 years really worth?

 Assume a 7% interest rate.

PVA =  = 

Excel: Nper = 20

 Rate = 7

 PMT = -50,000

 Solve for PV = 529,700.71

But, in the lottery, you get your first payment immediately and then the 19 others. In other words, you get each payment at the **beginning** of the year, rather than (as we usually do) at the end of the year. This is called an **Annuity Due**.

With Excel, enter “1” for type.

Annuity Due = Ordinary Annuity (1+r)

PVAD =  (1+r)

$529,700.71 (1.07) = $566,779.76

Note that it’s higher than when you get payments at the end of each year.

***Problem***: If interest rates are 4%, What is the present value of an ordinary annuity that pays you $10,000 per year for 10 years? That is – your first payment comes to you one year from today.

What is the present value if it is an annuity due – meaning you get the first payment today?

**Home Mortgages**: **Example**:

Amount of the Mortgage = $200,000

Obtain a 30 year mortgage at 6% APR

Monthly Payments

Solve for Pmt in excel

PV = 200,000

Rate = .06/12

Nper = 30\*12

Pmt = $1,199.10 = monthly mortgage payment

End of first month: Original Principal = $200,000

 Interest due = 6%/12 = .5% = .005

 .005 · $200,000 = $1,000

 You pay $1,199.10

 Interest 1,000.00

 Princ. $ 199.10

End of second month: New Princ. = $200,000 - $199.10 = $199,800.90

 Int. Due = .005 x $199,800.90 = $999.00

 You pay $1,199.10

 Interest 999.00

 Princ. $ 200.10

For 30 years, each month, the interest decreases and the principle increases. On the last month, you pay off all the principle with only a few dollars of interest.

Interest is due on the unpaid balance each month - not on the entire $200,000.

This is called **amortizing a loan**. Some money is paying the principal and some is paying the interest.

***Problem***: You borrow $30,000 to buy a new car. It is a four-year loan. The APR on your car loan is 7.5%. You make monthly payments with the first payment coming one month from today. What will your monthly payments be?

Any time we want to know what the payoff of our loan is, we merely have to look at the present value of the remaining future payments.

How much do we still owe on the mortgage above after 10 years of payments (20 years remaining)?

Solve for PV

Pmt = 1,199.10

Nper = 10\*12

Rate = .06/12

PV = $167,371.45

***Problem***: How much will you owe on your car loan (above) after 2 years?

**A Delayed Annuity**

What is the PV of $1,000 to be received each year for 10 years, starting five years from now if the discount rate is 7%?

 1000--------------------------------------------------------1000

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Note that you have to do this in two steps with Excel. First find the PV of the annuity as of time 4 (7,023.58). Then find the PV of that lump sum at time zero. Answer: $5,358.26.

PV = 

 = 7,023.58

 (1.07)4

 = 5,358.26

Whenever you do a two-step problem like this in Excel, you should always **reference the cell** with the number you want to input – don’t write it down and then type it in or you will have rounding errors.

***Problem***: How much money do you need to invest now to pay 4 years of college for your child if the first payment is due in 20 years, you estimate tuition to be $50,000 per year, and interest rates are 6% per year?

## Future Value of an Annuity

**Example**:

Invest $1,000 each month into a retirement plan for 35 years at 6% interest (APR). How much do you end up with?

Use the FV function in Excel

Pmt = 1,000

Rate = .06/12

Nper = 35 x 12

The answer is $1,424,710.30

FVA = 

FVA = $1,000  = $1,424,710.30

***Problem*:** If you save $30 per week by eating at home rather than eating out, and invest that money each week at an APR of 5%, how much will you have after 10 years? Assume exactly 52 weeks per year.

**Combining PV and FV into one Problem**

How much do you need to save each year for 10 years, so that you can pay for 4 years of college at $30,000 per year with an interest (investment) rate of 6%?

 30 30 30 30

\_\_\_\_\_C\_\_\_C\_\_\_C\_\_\_C\_\_C\_\_\_C\_\_\_C\_\_\_C\_\_C\_\_\_C\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

0 1 2 3 4 5 6 7 8 9 10 11 12 13

First solve for the amount you need at time 10. This is the PV of an annuity due

Pmt = 30,000

Rate = .06

Nper = 4

Type = 1

Answer = $110,190.36 at year 10

Then find out how much you need to put into the college fund each year so that it will grow and become $110,190.36 at year 10.

Solve for Pmt:

FV = 110,190.36

Rate = .06

Nper = 10

Answer = $8,359.92

***Problem***: What is your answer if you include two more years of graduate school?