The Time Value of Money

Chapter Outline

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Chapter Summary

Chapter Learning Objectives

After reading this chapter, students will be able to:

- Describe the concept of the time value of money.

- Calculate interest rates and compound interest.
- Describe the relationship between exponential growth and doubling time of investments or debt.
- Determine what happens to the value of a purchase or investment over time.
- Assess how a purchase or investment compares with other possible uses of your money.

INTRODUCTION: THE VALUE OF MONEY NEVER STANDS STILL

Benjamin Franklin famously said that "time is money." Franklin was warning about the loss of wages from skipping a day's work, but thinking of money in terms of time also applies to your personal finances. Most financial decisions involve exchanging sums of money across time. For example, when you save money, you put money aside now so you will have more of it in the future. When you borrow, you get money now and will have to pay back the amount you borrowed, plus an additional amount, in the future. The reason you have to pay back more than what you borrowed is that one dollar today is worth more than one dollar in the future. Why? Because that dollar can be invested (put to work) to earn more money.

To effectively deal with the financial decisions you face today, and those you will face in the future, you need to consider the **time value of money**: the idea that money available today is worth more than the same amount in the future due to its earning potential. When money is put to work, it earns **interest** over the course of a given time period. Interest is gained on the initial amount of money at a given rate of return. But there is more! The interest rate applies not only on the original amount of money but also on the interest earned on that money. Any interest that is earned on previously earned interest over several time periods is called **compound interest**. In order to make good financial decisions, you need to understand how compound interest works.

In this chapter, we will discuss the roles of interest rates and interest compounding in financial decision making, as well as how the value of money and your personal belongings change over time. Throughout the chapter, these concepts will be reinforced by a case study that illustrates the importance of the time value of money. With this basic knowledge and understanding, you will have an important tool to make sound financial decisions.

WHAT ARE INTEREST RATES?

Most financial transactions involve an exchange of money over time. When we save, we transfer money from today to the future. A person might save money for different personal financial goals (the importance of these goals was mentioned in chapter 1): to go to graduate school, to make a down payment on a house, to have enough money to live comfortably in retirement, or to take a great vacation. When we borrow money, we transfer money from the future to today. A person might borrow money again for a variety of personal goals: to pay for a college education, a house, or even a cup of coffee (if they pay with a credit card). Both borrowing and saving transactions involve interest rates. There are a number of ways to think of interest rates but the most basic is that the **interest rate** is the price of money.

When it comes to saving and investing, interest rates may also be thought of as *growth* rates. For example, when investors deposit a sum of money into a bank account or invest in the **financial markets**, places where financial securities (such as stocks and bonds) and other items of value are traded at prices that reflect supply and demand, they hope that the deposited sum of money will grow into a larger sum over time. The higher the interest rate, the faster and larger a sum will grow.

When people transfer money from the future to today, or borrow, the interest rate measures the *cost of borrowing*. Knowing the interest rate enables people to calculate how much they'll need to pay for the use of someone else's money. Yet another way to think of an interest rate is as the *rental price of money*. As discussed in chapter 1, when people borrow money, they are essentially renting it from a lender. The interest rate is the price paid for the money over and above the amount borrowed.

It's important to recognize that saving and borrowing are flip sides of the same type of transaction. When you deposit money at a bank, you are lending money to that bank. The bank is borrowing from you, which is why it pays you interest on the money in your account. Putting your savings in a bank, investing in a stock or bond, or lending money to someone are examples of making an investment that you hope will grow.

When you borrow money from the bank, the bank is the lender and you are the borrower, and you pay interest to the bank on the money you've borrowed. You will learn more about how banks work in chapter 4.

The Arithmetic of Interest Rates

How does time affect interest? An interest rate is measured with reference to a specific period of time. An interest rate of 2% *per month* is very different from an interest rate of 2% *per year*. For example, suppose you have $1,000 in the bank. If your interest rate is 2% per year, you have $1,020 at the end of one year. If the interest rate is 2% per month, you have that same amount—$1,020—at the end of one month.

Written as a formula, a simple interest calculation looks like this:

$$F = P\,(1 + r)$$

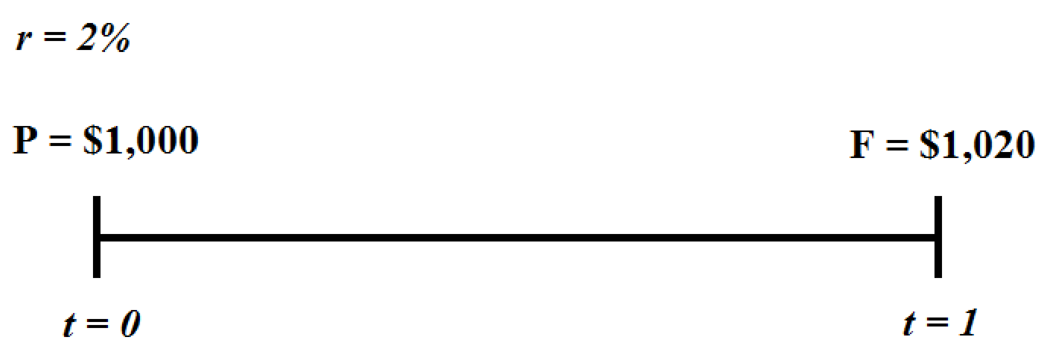
F represents the **final amount** (sometimes called the **future value**); P represents the

principal (or starting amount, also called the present value), and r represents the **interest rate**. In any formula, the letters, called **variables**, represent numbers that you know or that you are trying to figure out. If you know any two of these numbers, you can figure out the third number. Here, the $1,000 that you have invested is the principal (P), and the interest rate (r) is 2% per year, or .02. The final amount (F) is what you have after the interest is calculated. F is the value at a specified time in the future, assuming a certain interest rate. Once you know P and r, F is calculated as P times the quantity 1 plus r.

$$F = \$1{,}000\ (1 + .02)$$

After summing the quantities in parentheses, we have $F = \$1{,}000 * 1.02 = \$1{,}020$.

In other words, if you deposit money (P) at a bank at the beginning of the year, and the interest rate (r) you earn on that money is greater than zero, you will have more money (F) at the end of the year. How much more you have at the end of the period depends on the interest rate the bank pays. It can be helpful to picture this change in value on a timeline. At the time you invest the money (t = 0), you have $1,000, and after one year (t = 1), you have $1,020.



In personal finance, you always have to think of money in the context of time; in other words, think when you have the money and/or when money is due. In this particular case, you have a sum of money (P), which is equal to $1,000 today (t = 0). This sum will be $1,020 one year from now (t = 1). Why? Because it has earned interest (r) at the rate of 2 percent (r = 2%). The value of the sum of money that was $1,000 at time zero has become $1,020 at time 1. What would the value be at other time periods? This will be covered below.

It is important to visualize this concept for another reason as well. How can we make F as large as possible? F depends on two variables: P and r. To have a large sum of money at time 1, you have two options: start with a large P or find a high r.

Understanding and Calculating Compound Interest

Now we want to introduce another concept: **interest compounding**. Interest *compounds* over time; it builds on the interest earned on the initial amount. When you put money into an account that pays interest each year and you keep your money in that account for more than one year, you get extra interest in the second year, with respect to what you earn the first year, because the interest rate is applied not just on the principal, but on the principal plus the interest that is accumulating—or accruing—on that initial amount. In other words, interest builds on interest. For example:

Assume you invest $100 at a 20% interest rate per year for two years.

After the first year, your initial investment of $100 has grown to $100 plus 20% of $100, which can be written as

$100 + (.20 * $100) = $120.

The second year begins with a new balance of $120. The interest rate of 20% is then applied to this new balance and grows to

$120 + (.20 * $120) = $144.

Therefore, the total amount of interest on your initial investment is $20 + $24 = $44 and *not* \$20 + \$20 = \$40. The larger amount of interest (\$24 versus \$20) earned in the second year is because interest is earned not just on the initial amount (\$100) but also on the accrued interest in the first year (\$20).

A formula for the compound interest calculation is simply an extension of what we have learned previously:

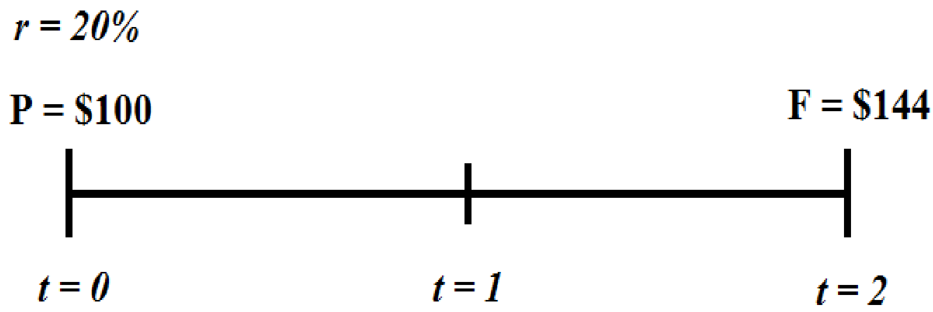
$$F = P\ (1 + r)^n$$

As before, the variable P represents the principal (or present value), r represents the interest rate per time period (expressed as a decimal), n represents the number of time periods, and F represents the future value. This is the formula we used before, but with an added variable (n) to represent the number of time periods involved in the interest compounding calculation. With this formula, if you know any three of these numbers, you can figure out the fourth number. To solve the problem of how much \$100 would grow over two years at a 20% annual rate of interest, replace the variables in the formula with the numbers you have:

$F = 100\ (1 + .20)^2$.

The calculation to find F—the amount you will have at the end of the two years—can be done a number of ways, all of which are based on algebra: on a handheld financial calculator, with an online financial calculator or smartphone app, or with a spreadsheet. But no matter how you do the math, understanding this formula and the information it gives you about the time value of money is enormously valuable. Each calculation method is explained in detail in the Resources section, and if you are new to these calculation methods, time value of money tables are provided so that you can check your work.

Here is a way to visualize this problem on a timeline. Note that the sum is due not at time t = 1 but t = 2.



Now we can ask the same question as before: What contributes to making F as large as possible? F depends on three variables: P, r, and n. To have a large sum of money at a future time, you have three options: start with a large P, find a high r, or invest for a large n.

To get a sense of the algebra at work behind all of these calculations, see Understand the Math 2-1.

To determine the future value in the above calculation using an online financial calculator, you would plug in the values as shown here, being sure the calculator is set to calculate the compounding annually:

Variable	**Value**	**Compute**
Present value (P)	100	PV
Future value (F)	?	FV
Number of periods (n)	2	N
Payment amount (A)	0	PMT
Interest rate per period (r)	.20	ir
Payment at:	☑ Beginning ☐ End	Clear

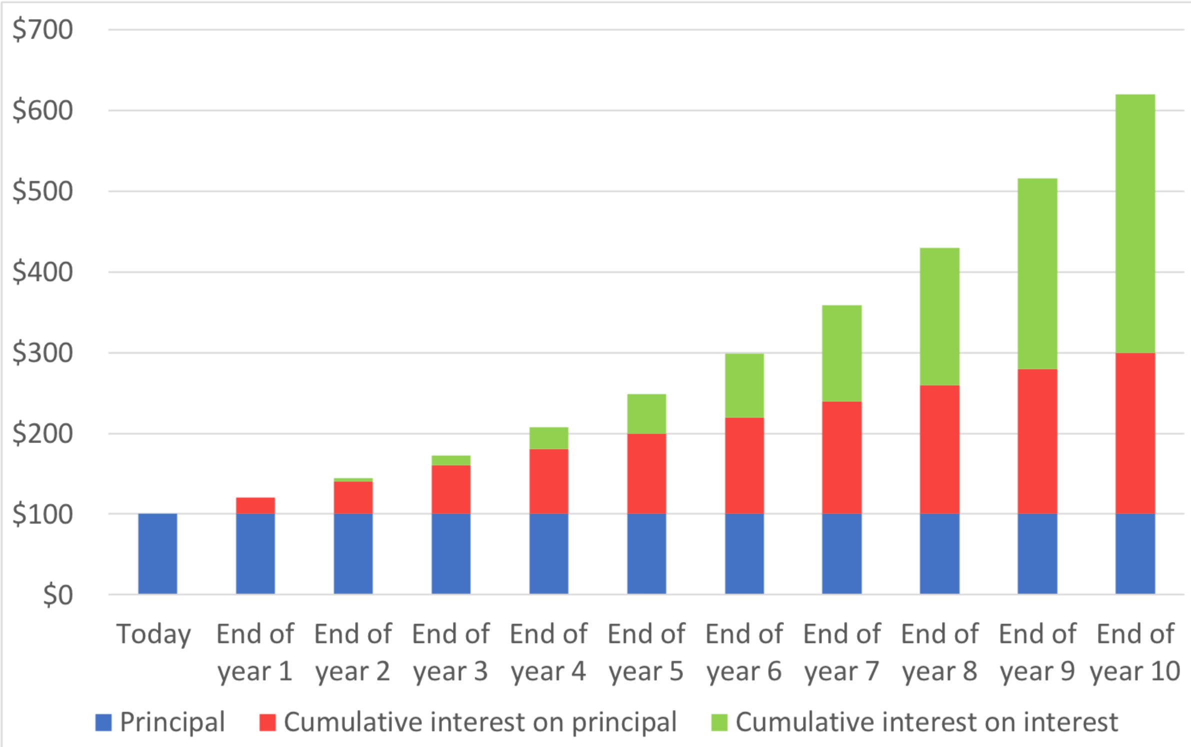
There are a variety of customized online calculators and smartphone apps that give you the same information as a financial calculator, though they are customized so you can select the calculator specifically designed for your situation: investing, using a credit card, saving, debt management, and so on. See the Resources section for more information.

A spreadsheet is an excellent way to do this calculation and allows for easy manipulation of the variables, for creating a month-by-month table of values, and for conversion of the calculation into charts, graphs, and tables. A pre-programmed spreadsheet for calculating the future value as interest compounds is available for download on the companion website. If you are new to spreadsheets, you can also find a description of how to work with them in the Resources section.

[Insert Understand the Math 2-1: Who Wants to Be a Millionaire here]

The power of interest compounding is illustrated in Figure 2-1. The blue represents your original investment. Every year, your original $100 earns an additional $20, pictured in red, which accumulates over the span of multiple years. The green bar represents the additional interest that is earned though compounding and shows how this interest grows.

Figure 2-1 Compounding Interest on $100 at 20% for Ten Years

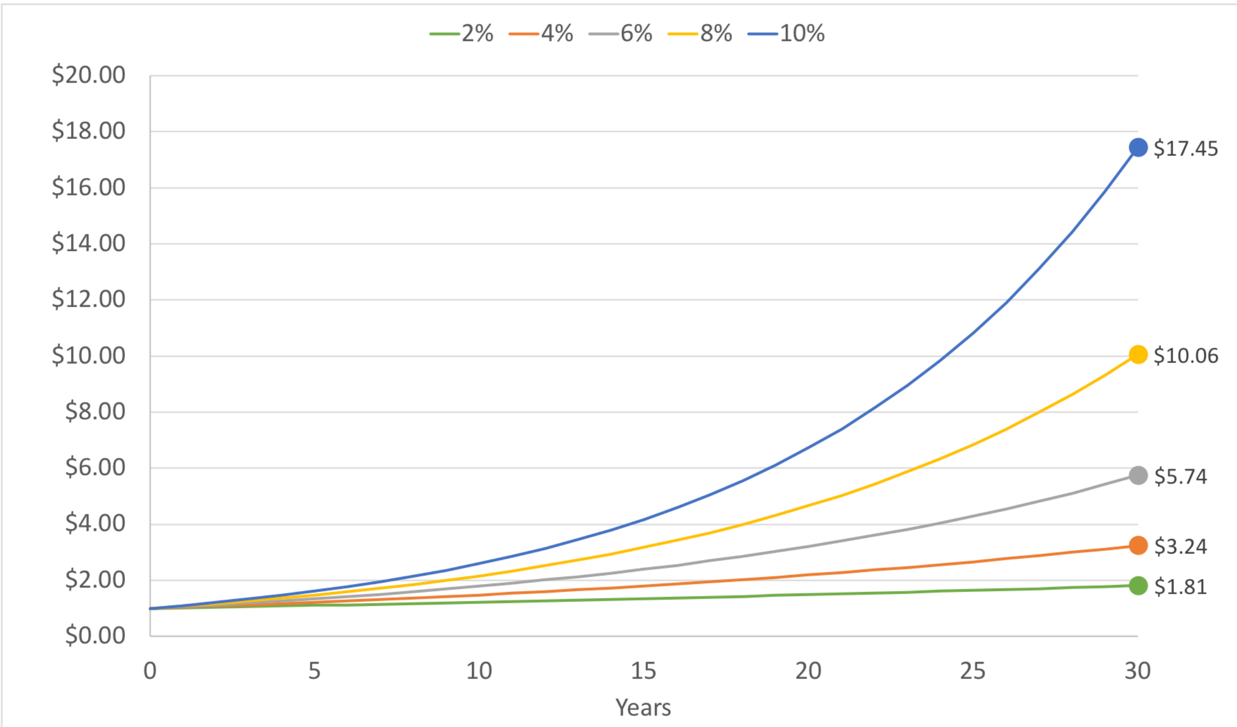


The extra $4 earned in the second year is the interest on the $20 worth of interest in the first year. This is interest compounding. The total interest accrues not only on the principal, but also on the interest. As demonstrated by each year's bar, interest on interest increases over time and eventually outweighs the interest on principal.

Figure 2-2 shows what happens with money invested over time at different interest rates. It grows because it is earning interest over time. The importance of time becomes evident when you calculate and compare how much you could have in savings by age 35 if you started saving at age 20, age 25, or age 30. The formula teaches us an important lesson about saving for the future: It pays to start saving early so that time (n) works in your favor! Moreover, the interest rate at which you invest matters a great deal. Over a span of 30 years, money invested at an interest rate of 2% almost doubles, while if you invest over that time span at an interest rate of

10%, it increases more than 17 times. This again shows the importance of knowing how to invest and the difference a higher interest rate can make.

Figure 2-2: Value of $1 Over 30 Years at Different Interests Rates



[Insert Mistakes People Make 2-1: Ignoring the Interest Rate here]

[Begin Case Study Part 1]

Case Study Part 1: Nick Receives an Unexpected Windfall and Considers His Options

Nick is a recent college graduate with a steady job, earning $51,000 annually. He is able to pay all his bills (rent, insurance, and utilities) and make his monthly student loan payment but has no savings apart from some occasional leftover money. Nick recently moved into a new apartment, this time without roommates, and he is looking into furnishing it. His dream is to do some travelling abroad. He is comfortable with his financial situation, but worries about his student debt.

Nick has the following three financial goals:

- Pay down his $20,000 student loan
- Accumulate $3,500 to cover the cost of a two-week vacation in Australia.
- Buy furniture for his new apartment

When Nick files his income tax return, he learns that he is going to get a $2,000 tax refund. It has also been a good year for his company, and Nick receives a $1,000 bonus. Nick thinks about what he should do with this unexpected $3,000.

Nick is very happy about his windfall because now he has an opportunity to pursue his goals. He considers all of them in turn, starting with the vacation in a country like Australia, which he has been dreaming about since college. The windfall is making him very close to having the money he needs to fly to Sidney and spend two weeks in Australia. If he shortens his stay there, he could go this year and not wait until he puts together the amount he has calculated a two-week vacation will cost.

Nick evaluates what he has to give up to do so. If he chooses the vacation, he will have to live in the new apartment for a while with the furniture he has. This means being less comfortable in his new place, continuing to use plastic chairs and his old bed, and not having a dinner table. If he goes to Australia, he cannot pay down his student loans or set money aside to make sure he makes regular payments, something he has been worrying about because what he earns now is just enough to cover his bills plus his student loans payments. These are the options he faces and he has to weigh them.

Discussion Questions

1. There are opportunity costs of pursuing one decision versus another. What other costs would you consider in addition to those listed in the text?
2. How does the decision to put the $3,000 toward his vacation goal impact Nick's present finances? His future finances? What risk is he taking by doing this?
3. Suppose Nick decides to go on vacation in Australia. Do you think that would be a wrong decision? Why or why not?

[End Case Study Part 1]

EXPONENTIAL GROWTH

What does it mean for a quantity to grow exponentially? **Exponential growth** is what happens when the rate of growth is proportional to (equal to a multiple of) the current amount. Any time you hear people talking about interest compounding at a certain rate, as in this chapter, you are considering exponential growth.

An investment of $100 that grows at 5 *dollars* per year is not an example of exponential growth, because the increase ($5) does not depend on the amount of investment ($100). But an investment that grows at 5 *percent* per year is growing exponentially. Table 2-1 contrasts what happens to an investment of $100 if it grows at (A) $5 per year, or (B) 5% per year.

Table 2-1: An investment growing at $5 per year versus 5% per year over 8 years

	Investment A **$5 per year**	**Investment B** **5% per year**
Start	$100	$100
Year 1	$105	$105

Year 2	$110	$110.25
Year 3	$115	$115.76
Year 4	$120	$121.55
Year 5	$125	$127.63
Year 6	$130	$134.01
Year 7	$135	$140.71
Year 8	$140	$147.75

What would happen after 30 years? The value of Investment A would increase to $250: $100 + 30 * $5 = $250. Investment B would have increased by 5% 30 times, so in 30 years its value would be:

$$F = (1.05)^{30} * \$100 = \$432.19.$$

This is a much higher amount and it speaks of the importance and effects of exponential growth.

Exponential Growth and Debt

Exponential growth calculations can also show us what happens over time to **debt**—money that has been borrowed. We will cover this topic in more detail in future chapters, but it is useful to mention here because many types of debt charge high interest rates; thus, exponential growth will make debt grow fast too!

With debt, the amount of money you owe grows because interest accrues on the borrowed amount over time. Let’s consider what happens over time when you spend money that you don’t have. Imagine you have borrowed $750 at a 10% annual interest rate and that you have agreed to pay it back after five years. Over the first year, the $750 debt grows by 10%, or

0.10 * $750 = $75.

As the borrower, you owe $750 + $75 = $825 at the end of the first year. But at the end of five years, your debt has grown exponentially because each year you pay interest not only on the principal, but also on the interest you owe from prior years. What you will owe is the future value (F) in the formula:

$F = P (1 + r)^n$.

What you borrowed ($750) is P; the number of periods (5) is n; and the interest rate (.10) is r. So the calculation is written like this:

$F = 750 (1 +.10)^5$.

You can use a financial calculator, an app, or a spreadsheet to determine the amount you will owe at the end of five years: $1,207.88.

How much would you owe after five years if you borrowed at a 3% annual interest rate? A 6% annual rate? A 15% annual rate?

Figure 2-3 shows how the future amount (F) grows *exponentially* as a function of time when you have different interest rates (r). The **exponential function** has many important properties. One property is the characteristic shape illustrated by Figure 2-3. Another is **doubling time**. Any exponential function of a quantity, in this case the invested money, doubles that quantity in the same amount of time. In other words, the amount of time it takes for an investment growing at a rate of 8% to go from $250 to $500 (nine years) is the same as the amount of time it takes that investment to go from $500 to $1,000 (an additional nine years).

Figure 2-3 How Rates of Interest Compound

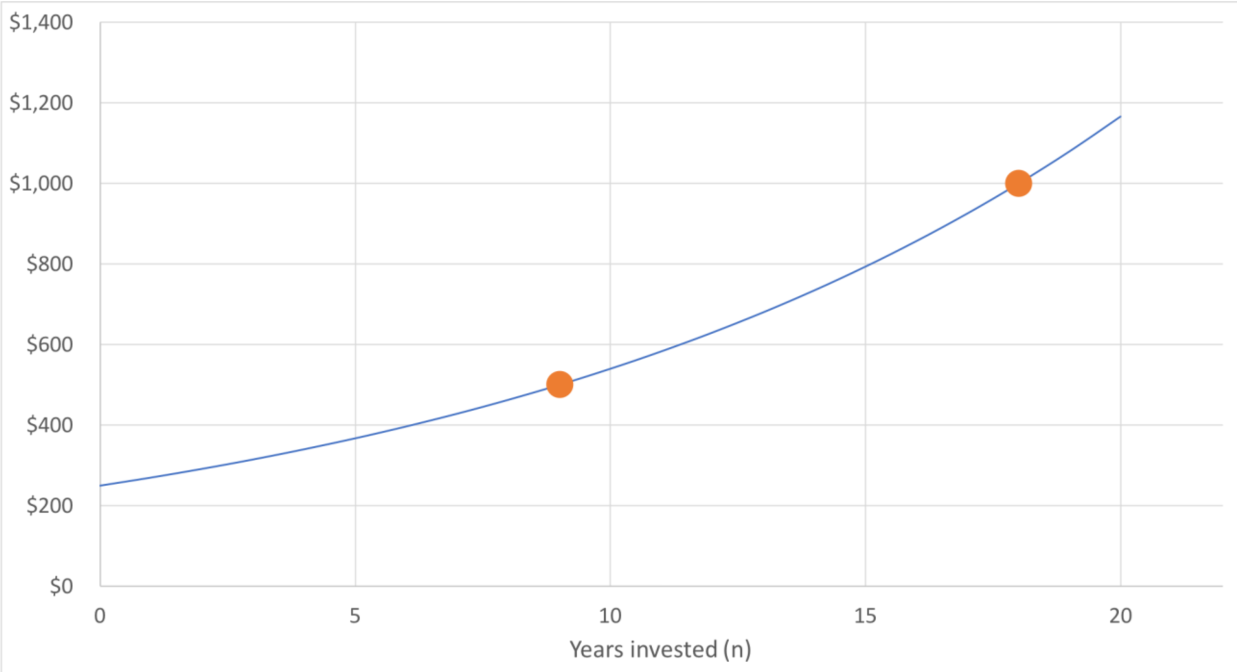


Figure legend: The horizontal axis shows time (n) and the vertical axis shows dollar amount. The line labeled X shows an investment of $250 growing at 8%, the line labeled Y shows an investment of $250 growing at 2%, and the line labeled Z shows an investment of $250 growing at .2%. The yellow dot indicates the first doubling period, at 9 years. The orange dot indicates the second doubling of that investment, at 18 years. It will take more than 20 years for the lower-interest investments to double.

The Rule of 72

Calculations can be pretty complex when considering interest compounding but there is a simple shortcut to determine how long an investment will take to double if you know the annual interest rate. It is called the **Rule of 72**, and it says that the number of years it takes to double your initial money (X) times the annual rate of return on your money (Y) equals 72:

X * Y = 72.

Thus, if you want to know the number of years it takes for your money to double, just divide 72 by the annual rate of return (as a whole number). The answer is an approximation, but gives you a good idea of the outcome without doing any complex calculations.

For example, suppose you can get an expected rate of return of 10% per year on an investment (of any amount). Assuming the interest rate remains at 10%, how many years would it take for your money to double?

X * 10 = 72

Dividing both sides by 10, we have:

X = 72/10 = 7.2

With a 10% rate of return, it takes 7.2 years for your money to double.

If the interest rate is lower, say 8%, it will take longer for the money to double. In this case, it will take 9 years (72/8 = 9). If you can get a much higher interest rate, it will take less time for your money to double: with a 15% interest rate, it would take only 4.8 years.

You can also use the Rule of 72 to determine the interest rate implied by an investment. If someone promises you that a project you invest in will double your money in 6 years, what interest rate is the project implicitly offering?

6 * Y = 72

Dividing both sides by 6, we have:

Y = 72/6 = 12

With a six-year doubling time on your investment, you are getting a rate of 12%.

With this shortcut, you can see that because it takes only 7.2 years for money to double if invested at 10%, you could have a *lot* of money if you start to save early. Suppose you start at age 20 and you save for 36 years at an interest rate of 8%. In that time, your money will grow five-fold!

The Rule of 72 can also help you quickly calculate how long it will take debt to double. If you have $7,000 in student loan debt and that money is borrowed at a 6% interest rate, it will take 12 years for that $7,000 in debt to become $14,000: 72/6 = 12. And if you have a credit card debt of $7,000 at an annual interest rate of 20%, it takes only 3.6 years (76/20) for that credit card debt to double.

To see where the "72" comes from, see Understand the Math 2-2.

[Insert Understand the Math 2-2: Where Does the Rule of 72 Come From?]

[Begin Case Study Part 2]

Case Study Part 2: Nick Considers Investing His Windfall or Paying Down His Debt

Nick decides that he would rather wait until he has enough money to go for the full trip to Australia. He therefore starts thinking about ways to obtain the $3,500 he needs for the trip. Nick considers whether his $3,000 could earn enough interest so that he would have $3,500 two years from now. His current bank account earns him only .26% APR. Nick does some research to see if he can find better interest rate offers. He finds an online bank that offers 2% a year on a savings account, compounded monthly. Using the formula $F = P (1 + r)^n$ with P equal to $3,000; r equal to 2%/12 (.02/12); and n = 24, Nick determines the following:

Principal (P) = **$3,000**

End of second year: $(1 + 0.02/12)^{24}$ * $3,000 = **$3,122.33**

Nick remembers reading about mutual funds and stocks, which seem to offer much higher returns, in the range of 7-10%, because they are riskier. What if he could invest his $3,000 at a

7% annual interest rate (compounded yearly)? Nick does a similar calculation to that above and finds that his money could grow by $434.7 in just two years, bringing him very close to his goal of $3,500. However, Nick, being very risk averse, does not really like the idea of investing his money in risky stocks.

Nick also has student loans that charge interest, so he turns next to the option of paying down his student loan. If he used his $3,000 to pay down his student loan debt, how would that compare to saving or investing the money? Nick took out a student loan of $20,000 with an interest rate of 7% for 10 years. He makes a loan payment each month of $232.20, a portion of which goes toward the interest on the loan and a portion reduces the loan principal. His windfall of $3,000 could shave months to years off of the duration of the loan. The potential gains on other investments are close to the loan's fixed interest rate, but those rates are not at all guaranteed.

Discussion Questions

1. How would investing the $3,000 in a bank account or stocks or mutual fund impact Nick's present finances? His future finances? What risk is he taking by doing this?
2. How would putting the $3,000 toward his student loan impact Nick's present finances? His future finances? What risk is he taking by doing this?
3. Suppose Nick does find an investment that earns 7%, should he invest there rather than paying off his student loan debt? Why or why not?

[End Case Study Part 2]

THE VALUE OF MONEY AND POSSESSIONS CHANGES OVER TIME

It is important to consider how much your money can grow over time because the prices of goods and services that you buy change over time too. This change, which we introduced in chapter 1 and observe in the United States and many other economies, is called inflation. As mentioned in chapter 1, **Inflation** is the percentage change or rate at which the price of goods, such as groceries, clothes, computers—in other words, the goods we buy regularly—increase over time. Inflation is another reason why a dollar today is worth more than a dollar tomorrow. You can buy more with a dollar today than you will be able to buy with that same dollar one year from now.

Measuring Inflation

Inflation is like an interest rate but it measures how quickly the cost of goods and services increase over time. Like interest, price changes compound during a given time period, such as a year or a quarter of a year. For example, suppose inflation is 10% and milk costs $1.00 a gallon. How much will a gallon of milk cost at the end of two years? We can use the formula $F = P (1 + r)^n$ to determine that:

Cost of milk after the first year = $1.00 * (1 + 0.1) = $1.10

Cost of milk after the second year = $1.10 * (1 + 0.1) = $1.21

Suppose you spent about $2,000 on clothes this year. With inflation at 2% per year, how much would it cost you to purchase the same amount of clothes in four years? How about in ten years?

You can use the same interest compounding formula:

$F = P (1 + r)^n$,

to determine that at 2% inflation (r), \$2,000 worth of clothes today (P) would cost \$2,164.86 (F) in four years (n):

$\$2,000 * 1.02^4 = \$2,164.86$.

In ten years, the clothes would cost:

$\$2,000 * 1.02^{10} = \$2,437.99$.

Note that inflation too grows as an exponential function. We can also use the Rule of 72 to figure out when the price of goods will double. If inflation is a steady 10%, it will take 7.2 years for the price of a gallon of milk to double.

Why is understanding inflation important to our financial decisions? If there is inflation, the amount that our money can buy over time decreases because of the increase in prices. In order to make sure we are not getting poorer over time, we need to grow our money at least at the same rate as inflation. Thus, keeping savings in cash, or in an account that earns zero interest, is not a good or even safe strategy. Not only do we run the risk that the money will be spent or stolen, but it will buy less tomorrow and even less the day after tomorrow. Future inflation rates are uncertain, so we inevitably face some risk when considering how much inflation will be down the line. Ultimately, we need to invest our money so that it can earn more (hopefully much more) than inflation.

Depreciation

If the value of money changes over time, what happens to the value of things you've bought over time, such as a sofa or a car? **Depreciation** refers to the process of material things losing value over time. Thus, we need to consider not just appreciation (increases in value), but also depreciation (decreases in value). They will follow the same rules of compounding explained above.

Physical assets usually have a depreciation component. They are made of things like wood, cotton, and metal (think of houses, clothes, and cars), all of which can depreciate over time. Physical objects like these wear out, get old, and rust. In the end, most physical objects get used up in one way or another and their value decreases.

Let's consider a possible depreciation scenario for a $3,000 sofa over the course of ten years. While there is no precise method for figuring out exactly what used furniture is worth, a sofa that cost $3,000 brand new could be worth less than $1,000 after a few years in its used condition. Using a standard depreciation rate of 20%, we can figure out how the value of furniture might change over time. We will subtract 20% of the value from $3,000 to estimate its worth as used furniture after one year:

$3,000 – .20 * $3,000 = .8 * $3,000 = $2,400.

In the second year, the depreciation is applied on the new value, not the original value, so here we have compounding of the depreciation rate:

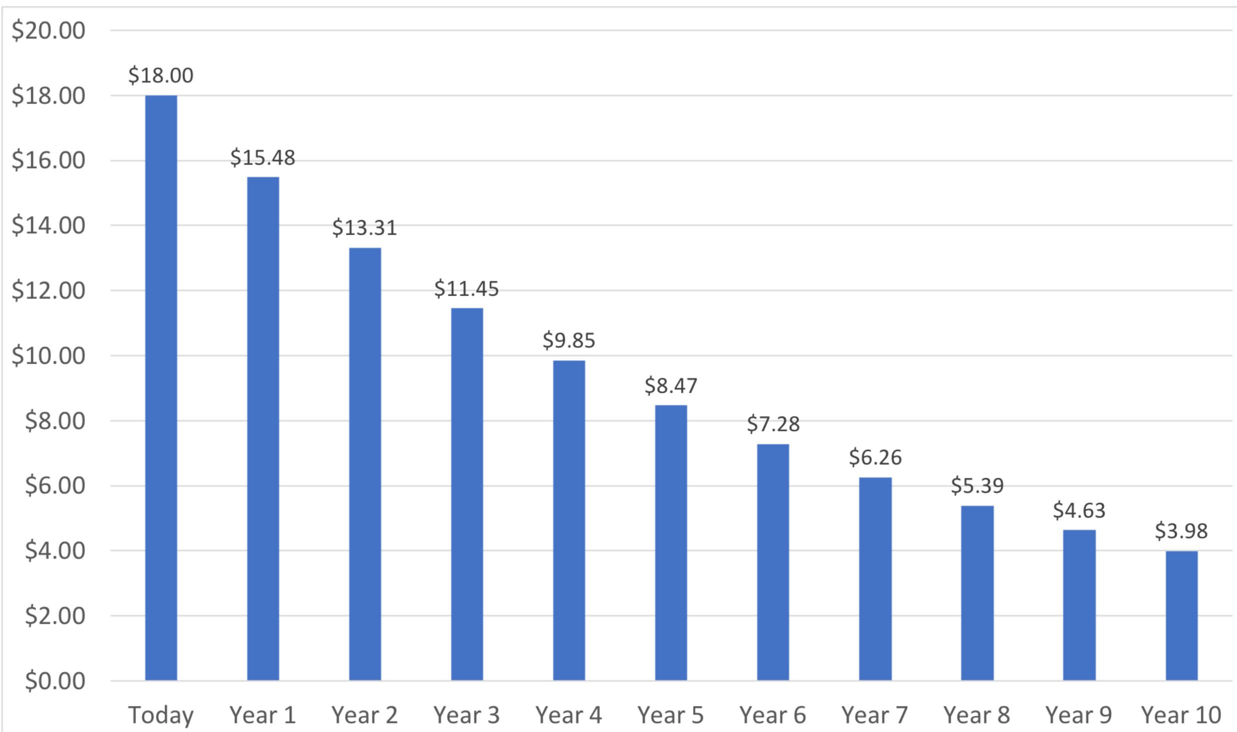
$2,400 – .20 * $2,400 = $1,680

Thus, in just two years, the sofa has lost almost half of its value. Instead of doubling in value every two years, as would be characteristic with exponential growth, the sofa's value is cut approximately in half every two years. The value of the sofa decreases exponentially over time. An exponential function describes the decreasing value of the sofa because the annual loss in value is proportional to the market value of the sofa at any given time.

Depreciation is an important consideration when you are making a big purchase, like furniture, electronics, or a vehicle (as will be discussed in chapter 7). You can use a financial calculator to do the math once you've determined the likely depreciation rate, and you can find some depreciation calculators online that will estimate loss in value of a variety of consumer items: furniture, vehicles, and so on. While depreciation is particularly useful to consider with items you will spend a lot of money on, it can be considered for any item you own.

Suppose you buy a used papasan chair for $18 and it depreciates by 14% per year. What will happen to the value of the chair over time? The graph below (Figure 2-4) shows how the chair's value depreciates over time, with it being worth very little after about ten years.

Figure 2-4: Change in Value of a Papasan Chair at 14% Depreciation



There is an important lesson here: while financial assets normally grow over time because of positive interest rates, many physical possessions depreciate over time. Their value goes down, not up, after they are bought, so be careful "investing" in these sorts of goods.

PRESENT VALUE

If you know how much money you will need in the future, how can you figure out how much you need to save today in order to have that amount at a specific point in time? For example, if you want to have $10,000 in three years for the down payment on a house, how much should you save today? What you are trying to find is the **present value**, or the current value of a known (or desired) future sum of money, given a specified rate of return.

Recall the equation that we used to calculate compound interest and its four variables (P, principal; r, interest rate; n, number of time periods: F, final value); remember that if you know any three of the variables, you can figure out the fourth.

$$F = P\,(1 + r)^n$$

For example, if your goal is to have $100,000 in ten years and you know the interest rate you will get is 7% (or .07), you know F = $100,000, r = .07, and n = 10. In this case, you can use the formula to find P, which will be the principal you will have to invest now. In this reverse calculation, the principal (also called the present value, as noted before), can be calculated by rewriting the formula as follows:

$$P = \frac{F}{(1 + r)^n}$$

Computing the present value is the inverse of interest compounding calculations and is referred to as **discounting**. From the present value formula, we see that the time value of money is the power of interest compounding in reverse. As a dollar is pushed farther into the future, n becomes larger, and its present value becomes smaller. The higher the interest rate (r), the more quickly the present value shrinks. Therefore, when we are discounting future values, the interest rate is often called the **discount rate**. You will sometimes hear the present value referred to as the “present discounted value.”

So, once you understand that this formula can be manipulated, you know that you can use it in two ways. You can use it to compute how much an amount today grows in the future or you can use it to determine how much you need to have today to reach a certain amount in the future. The difference between these two calculations is the unknown variable in the formula—in the first, F is unknown and in the second, P is unknown, but the formula is the same and it is critically important to master it to be able to make good financial decisions.

The following figures show the present value of the future sum of $110,000. If that $110,000 is to be received in one year and the interest rate is 10%, that amount is worth $100,000 today, as shown in Figure 2-5. But if that $110,000 is to be received in ten years, also at a 10% annual interest rate, its present value is just $42,410, as shown in Figure 2-6.

Figure 2-5: Present Value of $110,000 One Year from Now

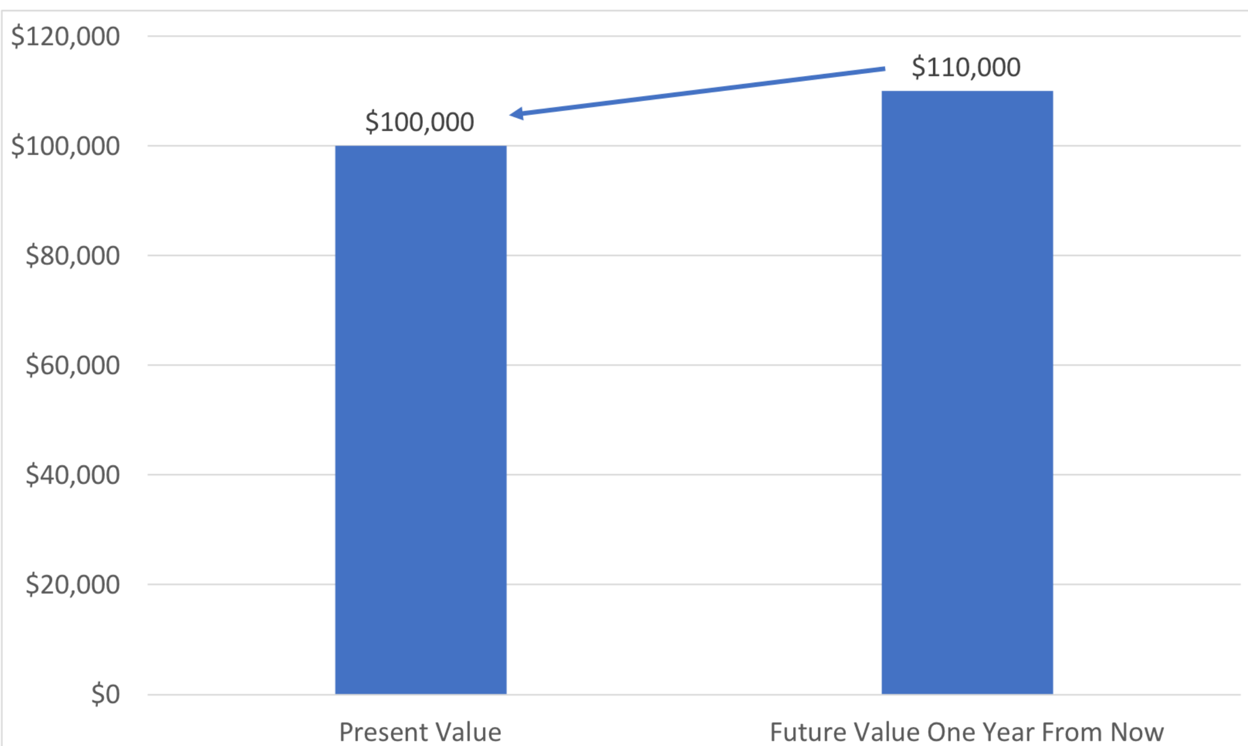
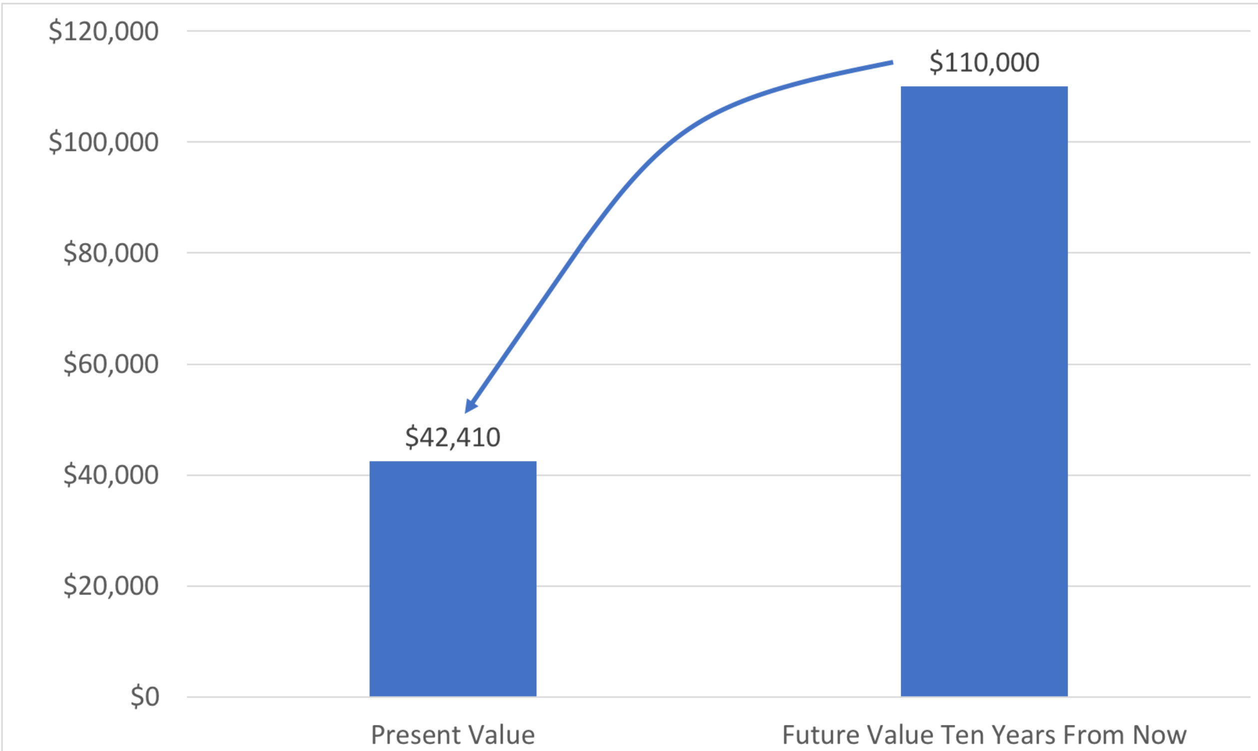


Figure 2-6: Present Value of $110,000 Ten Years from Now



[Begin Case Study Part 3]

Case Study Part 3: Nick Considers the Value of a Furniture "Deal"

Nick would also like to buy furniture for his new, but still rather empty, apartment and decides to shop around and see what sort of deals he can get.

He pays close attention to the advertised specials. He always shops at IKEA, but this time he goes out to the shopping mall close to his apartment to consider what they have in store. One store offers a "great deal" on a nice sofa with two chairs and a coffee table, enough to furnish his entire living room; the deal he is offered is no money down and no money due for one year, at which time $3,600 will be owed. Nick initially thinks, "I could take the furniture home now, put my money in the bank or invest it so to be able to pay the $3,600 in a year. But how much would he need to earn with a $3,000 initial investment to have $3600 in 1 year? A quick calculation on his smartphone shows that he would need to earn a 20% rate of interest in order to earn $600 in one year on a $3,000 investment.

A second store offers another “great deal”—this time for a sofa and easy chair set: no money down, no money due for two years, at which time $4,000 will be due. A salesman tells Nick that if he wants to pay for this same set of furniture now—a “cash and carry” offer—he can have it for $3,000.

What is the implicit interest rate associated with this “great deal”? Nick does the calculation: P is $3,000, the future value (F) is $4,000. And the time period (n) is two years. The implied interest rate on this “loan” comes to 15.5%, more than his student loan’s interest rate, or any of the other investments he’s contemplated so far. If he were to choose the option of paying $4,000 in two years for furniture that he could get for $3,000 today, he’d essentially have paid 15.5% interest for the privilege of putting off the payment, which he realizes would be a very expensive choice.

All of this shopping has convinced Nick that these so-called “great deals” are not so great after all.

Discussion Questions

1. Given that furniture depreciates over time, if you were to consider furniture as an investment, what would its return be over time?
2. If Nick were to spend $3,000 on furniture, how would that impact his present finances? His future finances? What risk would he be taking with this decision?
3. Explain how some of the “deals” offered by furniture shops are essentially loans to consumers. How do they compare with interest charge by credit cards? When would you

advise people to take these loans?

4. The salesman told Nick that if he couldn't make his payments, he shouldn't worry because he can always sell the furniture. What would you say about this salesman's statement?

[End Case Study Part 3]

[Insert Mistakes People Make 2-2: Misunderstanding the Relationship Between Savings and Debt]

WORKING WITH THE THREE QUESTIONS

In the course of this chapter, we have considered the ways in which saving and investing, debt, and spending are influenced by the fact that the value of money changes over time. Almost all financial decisions you make should be considered in light of this fact, and using the three questions when you are confronted with a decision will help you remember to do this:

1. How will this decision affect my present finances?
2. How will this decision affect my future finances?
3. What risk will I be taking with this decision?

1. How will this decision affect my present finances?

Spending money now affects your present finances by eliminating alternative spending options because money can't be used for two things at once. Spending money now also affects future finances because it takes away the option of investing the money for future use.

Asking yourself how a decision would affect your present finances can help you to think clearly about various options and your opportunity costs. For example, if you buy a new computer, given your fixed budget, you might give up the option of buying new sports equipment or using the money to invest. And investing would mean keeping your current computer for the time being.

2. How will this decision affect my future finances?

If you choose to invest your money, the rate of interest you obtain on that money will dictate how much wealth that investment will bring you in the future. The wealth resulting from investments at different interest rates can vary greatly. But because the prices of future goods and services also increase, whether you become richer or poorer over time depends on the rate at which you can grow your money. If the interest rate earned on investing is less than inflation, your investment can actually lose purchasing power over time.

In asking how decisions affect future finances, it is often useful to do some research and make calculations. Doing these calculations takes much of the guesswork out of your decision-making process.

3. What risk will I be taking with this decision?

One risk you take by spending money or investing it in a place you can't get to easily is the risk that you'll need the money right away. Some people address this risk by creating a **rainy day fund**—money available in case of an emergency. Moreover, because the future is uncertain, many of the variables we have considered here, such as interest rates and inflation, are uncertain, and we have to make decisions by considering a variety of possible scenarios, such as interest rates being lower than we expect or inflation being higher than we expect.

CHAPTER SUMMARY

Personal finance is personal. When faced with multiple options (for example, spending, investing, or borrowing), you need as much information as possible to decide which option is best for your circumstances. Understanding how money changes with time is a personal finance fundamental that you will come back to over and over again when you are making financial decisions.

- Saving and investing both matter if you are to grow wealth over time, and it pays to start saving early. Through interest compounding, the longer money is invested, the more rapidly it grows. You have to save money in order to have money to invest, but you have to invest it in order to make it grow, and the rate at which money grows and the time you can invest it both matter.
- The higher the interest rate, the more money compounds over time. You can calculate when an amount of money you've invested will double using the Rule of 72: the number of years it takes to double your initial money (X) times the annual rate of return on your money (Y) = 72.
- The value of money and goods do not stand still over time. Physical goods depreciate, so they lose value over time. Also, money decreases in value over time because the prices of goods you purchase increase over time. The percentage increase in prices is called the inflation rate.
- It's important to know how a purchase or investment compares with other possible uses of your money. Take time to do your research so you can fully understand your options, because each option comes with an opportunity cost. Personal finance is about making

good decisions. You will often be confronted with a variety of options and it's important to evaluate them against one another.

- You can figure out what a purchase or investment is actually worth by doing the math. You can't decide if a purchase, an investment, or a loan is good or bad unless you calculate the interest rate it earns or charges. A postponed payment option has a price, and the present value of a purchase might be more than it is worth. Not every deal is a good one. Doing the math helps us determine the true cost of something, whether you are paying a high interest rate, and what the future of your investment will be.

Also, note that the purpose of making good financial decisions is to have a better life. Having control of your money allows you to take care of business so that you can have fun without paying burdensome consequences. As explained in this chapter, problems can quickly compound to transform themselves into bigger problems.

KEY TERMS

Compound interest Interest calculated on principal (amount borrowed or invested) and on the interest that has accumulated in previous time periods.

Debt The amount of money borrowed by one entity from another.

Depreciation The reduction over time in the value of an asset due to wear and tear.

Discounting The process of calculating the present value of a payment (or payments) that will be received in the future.

Discount rate The interest rate used in calculating the present value of an amount of money to be had in the future.

Doubling time The period of time it takes for a quantity to double in value.

Exponential function In mathematics, a function whose value is a constant raised to the power of the variable.

Exponential growth Occurs when the growth rate of the value of a function is proportional to the function's current value.

Financial markets Any marketplace in which trading of securities, such as stocks and bonds, occurs.

Future value Value of a sum of money at a specific time in the future, assuming a certain interest rate.

Inflation A general increase in consumer prices and reduction in the purchasing power of money.

Interest The amount charged, expressed as a percentage of principal, by a lender to a borrower for the use of assets or the amount owed by a borrower to a lender. If you put money in the bank, it's the amount the bank (who is the borrower) pays to you (in this case, the lender) for the use of your money.

Interest compounding The process of interest being added to the principal (amount borrowed or invested) as well as any interest previously earned on that amount.

Interest rate The amount of interest to be earned or paid in a time period expressed as a proportion of the amount that has been invested or borrowed.

Present value The current worth of money that is to be paid at a future time at a specified rate of return. Also referred to as present discounted value.

Principal The amount borrowed or invested.

Rainy day fund Money available in case of an emergency.

Rule of 72 An estimation method for calculating the amount of time it will take an investment to double: doubling time multiplied by the annual rate of return equals 72.

Time value of money The idea that money available today is worth more than that same amount will be worth in the future.

Variables In mathematical formulas, variables are the letters that represent numbers you know or are trying to figure out.

CHAPTER HOMEWORK

Check Your Understanding

1. $100 in a bank account that offers 2% annual interest, compounded annually, will earn

   a. $2 every year
   b. more than $2 every year
   c. $2 in the first year and more in later years
   d. cannot be determined with this information

2. At a 2% rate of inflation, $100 kept under the mattress will be worth

   a. $100 in a year
   b. more than $100 in a year
   c. less than $100 in a year
   d. cannot be determined with this information

3. In the formula $F = P (1 + r)^n$, P stands for

   a. the amount invested
   b. the number of time periods
   c. the annual interest rate
   d. the final amount after n years of growth

4. The amount you would have to invest now at 5% interest in order to have $105 a year from now is called

a. the principal
b. the present value
c. $100
d. all of the above

5. If someone offers to sell something to you for $100 now or with “no money down and $200 due in a year,” the implied rate of interest is

a. 20%
b. 50%
c. 100%
d. 200%

Do the Math

1. A graduate student takes out a $5,000 loan that charges 12% interest per year. At the end of 4 years, how much will he owe?
2. For her fifth birthday, a girl’s grandparents set up an account of $1,000 earning an interest rate of 5% per year. When the girl withdraws the money in that account at age 20, how much will she receive?
3. An investor has a choice of two investment accounts paying 4% and 5% per year, respectively. If she were to put $1,000 into each investment account, how big a difference would there be between the two account balances in 5 years?
4. A woman is saving for a down payment on a house. If the down payment will be $30,000

and she can earn 6% interest annually on her savings, how much must she set aside today to make the down payment in 8 years?

5. A woman is saving for a $30,000 down payment on a house. She can earn 6% interest annually on her savings and sets aside $15,000 today. How long will it take for her savings to grow enough to make the down payment?
6. A consumer borrows $2,000 today. In two years, he must repay the lender $2,400. What is the implied annual interest rate on this loan?
7. A consumer borrows $500 at a 12% annual interest rate. Approximately, how long would it take this debt to double?
8. A person invests $10,000 in stock today and another $10,000 in five years. If the investment account earns an average of 8% per year for the first five years and an average of 10% per year for the following five years, how much will be in the account at the end of ten years?

Thinking Hard

1. Suppose you put $1,000 into an account earning 3% per year, for ten years. Suppose during that time the rate of inflation is 3%. At the end of 10 years, how will the purchasing power of your money compare with today?
2. Suppose you put $1,000 into an account earning 5% per year, for ten years. Suppose during that time the rate of inflation is 3%. At the end of 10 years, how will the purchasing power of your money compare with today?
3. Which is a better use of money, investing it at a 7% interest rate, or paying off a loan at an 8% interest rate? Why?

4. Which is a better use of money, investing it at an 8% interest rate, or paying off a loan at a 7% interest rate? Why?

Working with the Three Questions

Question 1: How will this decision affect my present finances?

Nate spent the summer after his first year of college working for a construction company. He was at work by 7 a.m. each day and home, exhausted, by 4 p.m. Spending so much time at work and having to go to bed early every night meant that he hardly had time or energy to spend the money he was earning. When he returned to college in the fall, he was excited to see his friends and use his summer earnings on activities such as mountain biking, skiing, and going out on the weekends. He had saved $3,000 and would probably be able to earn $600 working over winter break. He had an older mountain bike but really wanted one he could ride on more advanced trails. In mid-September, he found the perfect bike—last year's model on sale—for $2,700.

The previous year, Nate's parents had given him $200 per month in spending money (about $50 per week), but they will not able to do so this semester.

a. If Nate uses his savings to buy the bike, how much per week will he have left to use as spending money? (A semester is 15 weeks long.)

Question 2: How will this decision affect my future finances?

Nate decides to spend his savings on the bike, but just before heading to the bike shop, he learns that the deadline for purchasing a discounted student ski pass is the next week. The pass costs $500. His $3,000 won't cover both the bike and the ski pass, but he has recently obtained a credit card for "emergencies." The credit card has a 24 percent interest rate on any balance carried longer than a month. He decides to use the credit card to pay for the bike and to use his savings to cover the ski pass. He figures he can pay off the credit card balance by spending three weeks of his winter break working full time at his $15 per hour construction job.

Consider the implications of Nate's decision.

a. What will he owe on his credit card by the end of the semester, assuming an interest compounding as in the formula seen in the text?
b. How many hours will he have to work to pay it off?
c. How much will he have in his bank account when the second semester begins?
d. What might this mean for the following summer?

Question 3: What risk will I be taking with this decision?

Many financial risks stem from the assumptions people make about their own behavior, their future actions, and the consequences of those actions.

a. What assumptions would Nate have made that make his decision a reasonable one?

Think about his spending habits during both semesters, his employment during breaks, his other needs and desires during that period, and about his third year of college.

b. Analyze the financial risks involved if one or more of those assumptions proves to be wrong.

Practice Your Decision Making

Sophie is a recent college graduate of Michigan State University working full time as a software developer for a mid-size company in East Lansing, Michigan. She makes $51,000 per year. She has her own small apartment, which she is renting, a circle of friends she met in college, hobbies and activities that keep her happy, and her beloved cat.

Although she loves the job in this small research company run by one of her favorite professors, after a year of working there, she begins to notice the better rates of pay some of her friends are receiving for similar work. Two of her friends in particular are making $6,000 per year more than she is. They tell Sophie that they received higher pay after finishing a master's degree in the subject. Her employer confirmed that even in his small company, a master's degree in computer science would mean an instant raise of between $5,000 and $7,000. He encouraged Sophie to go back to school for such a degree, and even offered to let her continue working half time at the company to pay her way through school. Her lifestyle is simple and she can live on half her current pay. But she is well aware of the impact graduate school will have on her current finances. She has already answered the first question for financial decision making.

Sophie applied and was admitted to the Michigan State master's program, with a scholarship that covered tuition. Her parents, who value education above almost everything else, are very proud of her. They see this as a great opportunity for their daughter.

Sophie estimates that with half-time work keeping her busy, it will take two years to complete the program. Sophie enjoyed college and was an excellent student. She knows that she can easily do the work of the master's program. But her interest in the program is largely financial, as she isn't sure how much it will actually help her do her work. It seems entirely possible to learn everything necessary on the job. Sophie is wondering about the second question for financial decision making: How will this decision affect my future finances? The answer to this question will determine her decision about graduate school, at least for the time being.

Before accepting or rejecting the offer from Michigan State, Sophie has asked you to help her think through the financial implications of her decision.

Part 1: Do some research

Most salary raises are based in part on the rate of inflation. Look up recent inflation rates and estimate the rate of inflation that will impact Sophie's salary based on these. As Sophie is a good employee, add an extra percentage point to this rate to estimate a larger raise based on performance. How do you expect an extra $5,000 in salary to grow over the course of twenty years? As Sophie won't get the extra $5,000 for two years, what is the present discounted value of that sum?

Make a table comparing her annual salary for twenty years, with and without the extra education. At what point does her expected extra pay compensate her for the loss of two half-years of employment?

Is Sophie currently getting a competitive wage for her experience and job category in the East Lansing area? Do some research on salaries for entry-level software developers. Would Sophie be better off searching for a job with a different company? In a different geographic area?

One of Sophie's options is to keep her current job but save one-third of her salary and put it into an investment that will grow over time. Maybe this would be a better strategy than getting another degree. Look up some options for investments and choose one with a good rate of return. What would happen to the money saved in this manner over the course of 20 years if put into such an investment?

Part 2: Make a plan

Sophie has thought through how her decision to go back to school will affect her present financial situation. But she needs a clear picture of how it will affect her future financial situation. Describe in detail the financial consequences of four options:

1. keeping her present job,
2. keeping her present job but investing a third of her salary,
3. obtaining a degree in two years and going back to her present job with an estimated $5,000 raise,
4. and seeking a better paid position elsewhere.

In addition, Sophie has not even considered the possible financial or career risks of these decisions. Imagine the ways in which these decisions might not work out as planned, and outline the risks you see in each of them.

You Are Your Own CFO

Consider your present situation: Do you have any debt at the moment? (credit card, student loan, car?). What interest rate are you paying on that debt and how frequently does it compound? Do you have any savings or investments at the moment? (savings account or brokerage accounts) If so, what interest rate are you earning on those? Without divulging this information, write a short paragraph assessing your satisfaction with your current financial situation. Turn one copy in and keep a copy for yourself.

Think about your future: For each of the financial goals you set in chapter 1, estimate how much money you would have to have on hand to meet that goal, for example the price of a car or the down payment for a house. How much would you have to put aside today, at today's estimated interest rates, in order for it to grow to that amount by the time you need it? Do some research and compare what happens if you use interest rates offered by banks or any other investment you read about. Turn in a short essay describing your calculations and comparisons. Keep a copy for yourself.

CHAPTER 2 FEATURES

Understand the Math 2-1: Who Wants to Be a Millionaire?

If a 25-year-old invests $20,000 at 10% annual interest rate, how old will this investor be when the investment becomes $1,000,000?

Here F = $1,000,000, P = $20,000, r = .10 and the problem is to find n. You can use a financial calculator, a spreadsheet, or you could do algebra! The equation we need is:

$$F = P\,(1 + r)^n$$

Here we need to solve for n in terms of the other variables, which we know. Dividing both sides by P gives

$$F/P = (1 + r)^n.$$

Taking the logarithm of both sides gives

$$\mathrm{Log}\,(F/P) = \mathrm{Log}(\,(1 + r)^n) = n * \mathrm{Log}\,(1+r).$$

Solving for n gives

$$n = \mathrm{Log}\,(F/P)\,/\,\mathrm{Log}\,(1 + r).$$

Plugging the numbers into a financial calculator gives the answer. As you can see, it is quite a bit less work to use the calculator!

The answer is 66 years old. This is the magic of interest compounding, it is truly possible to become a millionaire during a lifetime, even with a single initial investment.

Understand the Math 2-2: Where Does 72 Come From?

It is important to know that the rule of 72 is just an approximation. Where does the "72" come from? From the familiar formula $F = P (1+r)^n$.

If the amount has doubled, then F is twice P and we have

$2P = P(1+r)^n$,

or equivalently,

$2 = (1+r)^n$.

Taking the natural logarithm (sometimes written Ln) of both sides gives

$Ln(2) = n * Ln(1+r)$

It is a very special mathematical fact that for small r, Ln(1+r) is approximately $r - .5 * r^2$, so we have then

$Ln(2) = n * (r - .5r^2)$

or

$Ln(2)/(1-.5r) = n * r$.

For interest rates between .05 and .12, the left hand side varies from .71 to .73, so .72 is a reasonable approximation. Converting the decimal expression of r into percent gives

$72 = n * r$,

or the Rule of 72.

Mistakes People Make 2-1: Ignoring the Interest Rate

One mistake that people commonly make is focusing on the amount they are saving over time but not paying attention to the rate of interest being earned on their savings. If you have a savings account, do you know what the interest rate is? The impact of the interest rate on savings can be substantial. Play around with some calculations to see how much of a difference it can make. Try calculating the amount of interest you would earn after 20 years on $1,000 invested at 2%, 5%, and 10% annual rates of interest. How do the results compare?

Mistakes People Make 2-2: Misunderstanding the Relationship Between Savings and Debt

People often think that they are breaking even if they have $5,000 worth of debt and the same amount, $5,000, in the bank. However, if the interest rates on each respective amount are different, they are not breaking even. Interest on debt is frequently higher than interest on savings, meaning debt generally costs people more than savings earns them. When deciding between saving and paying down debt, do the calculations to determine which is your better option.

Resources

Introduction to the financial calculator

The main equation used for calculating interest (whether on a loan or an investment) is:

$$F = P\,(1 + r)^n$$

P = principal, or present value

r = interest rate per time period (as a decimal)

n = number of time periods

F = final value

If you know any three of these four quantities, P, n, F, or r, you can figure out the fourth. You can solve for it directly using algebra, or you can employ a guess-and-check strategy to get an estimate. A simple financial calculator takes advantage of this simple algebra to do the calculation for you.

Financial calculators also have a fifth variable, called “payment amount” which allows you to compute what happens if you add the same amount of money to your investment at each time period (or equivalently, if you pay a certain amount on your loan at each time period.) The calculation required to take regular identical payments into account is more complicated and you will definitely want a calculator to do that. If you need to know more detail, such as how much of each payment on a loan goes to interest and how much to principal, or what would happen if you made different size investments each month, then it is better to use a spreadsheet such as the one provided at the textbook’s companion website.

Hand-held financial calculators usually have you enter a number then press one or more keys to identify which of the five variables it represents. Online calculators have various forms but usually have fields labeled by the five variables, in which you enter four out of five of the numbers. It then calculates the fifth one. Some computers come equipped with financial calculators, or you can download one from the web. There are also financial calculator apps that you can install on your smartphone. Below you will see the version of the financial calculator provided on the textbook’s companion web site. We will use this calculator as our examples throughout the text.

Variable	**Value**	**Compute**
Present value (P)		PV
Future value (F)		FV

Number of periods (n)		N
Payment amount (A)		PMT
Interest rate per period (r)		ir
Payment at:	☑ Beginning ☐ End	Clear

The compute buttons represent the five variables above: principal / present value (P) is PV, future / final value (F) is FV, number of periods (n) is N or NP, regular payment amount (A) is PMT, and interest rate per period (r) is ir. In addition, there is a clear button which removes all the numbers you have entered and lets you start over. The calculator tracks whether the money involved is being paid out by you or being received by you. Use negative numbers for owed balances or money paid into an account, and positive numbers for money to be withdrawn from an account or balances in your favor.

To use the calculator, enter four of the five variables in the corresponding blue boxes. The calculator assumes regular payments are made at each time period and you must indicate whether the payment is at the beginning or end of each period by clicking on one of those two buttons (even if the regular payment is zero). To compute the remaining number, click on the corresponding calculation button and the answer will appear in the remaining blue field.

Let's use a financial calculator to do a simple interest calculation. Say you want to know how much money you will have in an investment that has an annual interest rate of 12%. The account has a starting balance of $1,000.00 and you are not planning to make any further deposits. You know that your interest will be compounded monthly. You would like to know the value of this account in 10 years.

In this example, principal or present value (PV) would be set to –1,000. Why the negative? Because you are going to pay it into an account and it will be unavailable to you for ten years. The calculator will produce a final value (FV) that is positive, indicating the value of the return to you at the end of the ten-year period.

As interest is compounded monthly, you will have 120 periods (N) in your calculation. No payments will be made during these periods, so the regular payment amount (PMT) is set to zero. It doesn’t matter whether you choose the beginning or end button, but you have to pick one.

For the interest rate per period (per month in this case), you divide the annual interest rate of 12% by 12 = 1% per month. Enter this in the blue field as .01.

It should be noted that technically, 12% per year does not equate to 1% per month. Why? Because of the *time value of money*. The 1% you pay in January is worth slightly more than the 1% you would pay in December of that same year. By paying 1% per month, you are actually paying more than 12% paid at the end of the year, but less than 12% paid at the start of that same year. The difference, however, is small, so banks and other financial institutions just approximate the whole process by dividing the annual rate by 12.

When all of the numbers have been entered into the calculator, it will look like this:

Variable	**Value**	**Compute**
Present value	1,000	PV
Future value		FV
Number of periods	120	N
Payment amount	0	PMT
Interest rate per period	.01	ir
Payment at:	☑ Beginning	Clear

☐ End	

The only field left empty is that of future, or final value (F). Push the FV button and you will see a number appear: 3,300.39. This is the amount available to you after ten years of monthly compounding at 1% per month.

If you wanted to describe borrowing $1,000 at 12% per year compounded monthly, to be paid in full at the end of 10 years, you would do a similar calculation. This time you would enter a positive 1,000 as the present value because it is available to you now. The calculator would then return -3,300.39 as the final value, indicating that you would have to pay that amount at the end of ten years.